APPENDIX V

DEVIATION FROM UNIFORM STRAIN

In Chapter III, the assumption was made that a polycrystalline material subject to macroscopic uniaxial strain realizes microscopic uniform strain, where macroscopic and microscopic are relative to the grain size. This is not strictly true. In individual crystallites deviation from uniform strain can be expected. The purpose of this appendix is to obtain a measure of this deviation from uniform strain in cubic polycrystalline material.

If a polycrystalline material is subject to macroscopic uniaxial strain and further constrained to uniform strain in each crystallite, then the associated elastic energy is

$$E_V = \frac{1}{2} m_V e^2$$

where

$$m_V = K_V + \frac{4}{3} \mu_V \tag{V.1}$$

is the longitudinal modulus obtained from the Voigt assumption.[36]

$$K_V = \frac{1}{3} (c_{11} + 2c_{12}) \tag{V.2}$$

and

$$\mu_V = \frac{1}{5} ((c_{11} - c_{12}) + 3c_{44}) \tag{V.3}$$

are the bulk modulus and shear modulus in terms of the elastic stiffness coefficients.

If a polycrystalline material is subject to macroscopic uniaxial strain but constrained locally to uniform stress, then the elastic energy is

$$E_R = \frac{1}{2} m_R e^2$$

where

$$m_R = K_R + \frac{4}{3}\mu_R \qquad (V.4)$$

is the longitudinal modulus obtained from the Ruess assumption.[37]

$$K_R = \frac{1}{3}(c_{11} + 2c_{12}) \qquad (V.5)$$

and

$$\mu_R = \frac{5}{4(S_{11} - S_{12}) + 3S_{44}} \qquad (V.6)$$

are the bulk modulus and shear modulus in terms of the elastic stiffness and compliance coefficients.

In the actual case, the elastic energy is

$$E = \frac{1}{2} m e^2$$

where m is some unknown longitudinal modulus. It has been proved that[64]

$$m_R \le m \le m_V .$$

Experiment has shown that m is very close to the arithmetic average of the Voigt and Ruess approximations;[35]

$$m \simeq \frac{m_R + m_V}{2} . \qquad (V.7)$$

Since